# DP IB Maths: AA HL

Your notes

## 2.7 Polynomial Functions

### Contents

## 2.7.1 Factor & Remainder Theorem

Your notes

# Factor Theorem

### What is the factor theorem?

- The **factor theorem** is used to find the linear factors of **polynomial** equations
- This topic is closely tied to finding the **zeros** and **roots** of a **polynomial** function/equation
  - As a rule of thumb a **zero** refers to the polynomial function and a **root** refers to a polynomial equation
- For any **polynomial** function $P(x)$
  - $(x - k)$ is a **factor** of $P(x)$ if $\mathbf{P(k) = 0}$
  - $\mathbf{P(k) = 0}$ if $(x - k)$ is a **factor** of $P(x)$

### How do I use the factor theorem?

- Consider the polynomial function $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ and $(x - k)$ is a **factor**
  - Then, due to the factor theorem $P(k) = a_n k^n + a_{n-1} k^{n-1} + \ldots + a_1 k + a_0 = 0$
  - $P(x) = (x - k) \times Q(x)$, where $Q(x)$ is a **polynomial** that is a factor of $P(x)$
  - Hence, $\dfrac{P(x)}{x - k} = Q(x)$, where $Q(x)$ is another factor of $P(x)$
- If the linear factor has a **coefficient of x** then you must first factorise out the coefficient
  - If the linear factor is $(ax - b) = a\left(x - \dfrac{b}{a}\right) \rightarrow P\left(\dfrac{b}{a}\right) = 0$

### Examiner Tip

- A common mistake in exams is using the incorrect sign for either the root or the factor
- If you are asked to find integer solutions to a polynomial then you only need to consider factors of the constant term

Your notes

### Worked example

Determine whether $(x-2)$ is a factor of the following polynomials:

a) $f(x) = x^3 - 2x^2 - x + 2$.

Step 1: Determine k

Our linear function is $x - 2$

→ so $k = 2$

Step 2: Apply factor theorem

For $x - 2$ to be a factor of $f(x)$,

$f(2)$ has to equal zero

$$f(2) = (2)^3 - 2(2)^2 - (2) + 2$$
$$= 8 - 8 - 2 + 2$$
$$= 0$$

$f(2) = 0$,
so $x - 2$ is a factor of $f(x)$

b) $g(x) = 2x^3 + 3x^2 - x + 5$.

Your notes

Step 1: Determine k

Our linear function is $x - 2$

→ so $k = 2$

Step 2: Apply factor theorem

For $x - 2$ to be a factor of $g(x)$, $g(2)$ has to equal zero

$$g(2) = 2(2)^3 + 3(2)^2 - (2) + 5$$
$$= 16 - 12 - 2 + 5$$
$$= 7$$

$g(2) = 7$, so $x - 2$ is not a factor of $g(x)$

It is given that $(2x - 3)$ is a factor of $h(x) = 2x^3 - bx^2 + 7x - 6$.

c) Find the value of $b$.

Your notes

Step 1: Determine k

Our linear function is $2x-3$

$\rightarrow$ so $k = \frac{3}{2}$

Step 2: Apply factor theorem to find b

Since $2x-3$ is a factor of $h(x)$,

$h\left(\frac{3}{2}\right) = 0$

$$0 = 2\left(\frac{3}{2}\right)^3 - b\left(\frac{3}{2}\right)^2 + 7\left(\frac{3}{2}\right) - 6$$

$$= \frac{54}{8} - \frac{9}{4}b + \frac{21}{2} - 6$$

$$b = 5$$

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## Remainder Theorem

### What is the remainder theorem?

- The **remainder theorem** is used to find the remainder when we divide a **polynomial** function by a linear function
- When any polynomial $P(x)$ is divided by any linear function $(x - k)$ the value of the remainder $R$ is given by $P(k) = R$
  - Note, when $P(k) = 0$ then $(x - k)$ is a factor of $P(x)$

### How do I use the remainder theorem?

- Consider the polynomial function $P(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$ and the linear function $(x - k)$
  - Then, due to the remainder theorem $P(k) = a_n k^n + a_{n-1}k^{n-1} + \dots + a_1 k + a_0 = R$
  - $P(x) = (x-k) \times Q(x) + R$, where $Q(x)$ is a **polynomial**
  - Hence, $\frac{P(x)}{x-k} = Q(x) + \frac{R}{x-k}$, where $R$ is the remainder
- If the linear function has a **coefficient of *x*** then you must first factorise out the coefficient
  - If the linear function is $(ax - b) = a\left(x - \frac{b}{a}\right) \rightarrow P\left(\frac{b}{a}\right) = R$

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## Worked example

Let $f(x) = 2x^4 - 2x^3 - x^2 - 3x + 1$, find the remainder $R$ when $f(x)$ is divided by:

a) $x - 3$.

Step 1: Determine k

Our linear function is $x - 3$

→ so $k = 3$

Step 2: Apply remainder theorem

$f(3) = R$

$f(3) = 2(3)^4 - 2(3)^3 - (3)^2 - 3(3) + 1$

$f(3) = 162 - 54 - 9 - 9 + 1$

$f(3) = 91$

$R = 91$

b) $x + 2$.

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Step 1: Determine k

Our linear function is $x + 2$

$\rightarrow$ so $k = -2$

Step 2: Apply remainder theorem

$f(-2) = R$

$f(-2) = 2(-2)^4 - 2(-2)^3 - (-2)^2 - 3(-2) + 1$

$f(-2) = 32 + 16 - 4 + 6 + 1$

$f(-2) = 51$

$R = 51$

The remainder when $f(x)$ is divided by $(2x + k)$ is $\frac{893}{8}$.

c) Given that $k > 0$, find the value of $k$.

Step 1: Apply remainder theorem

$2x + k = 2\left(x + \frac{k}{2}\right)$ $\quad f\left(-\frac{k}{2}\right) = \frac{893}{8}$

$\frac{893}{8} = 2\left(-\frac{k}{2}\right)^4 - 2\left(-\frac{k}{2}\right)^3 - \left(-\frac{k}{2}\right)^2 - 3\left(-\frac{k}{2}\right) + 1$

Step 2: Solve for k using your GDC

$k = 5$

Your notes

# 2.7.2 Polynomial Division

Your notes

## Polynomial Division

### What is polynomial division?

- Polynomial division is the process of **dividing two polynomials**
  - This is usually only useful when the **degree of the denominator** is **less than or equal** to the **degree of the numerator**
- To do this we use an algorithm similar to that used for **division of integers**
- To divide the polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ by the polynomial $D(x) = b_k x^k + b_{k-1} x^{k-1} + \ldots + b_1 x + b_0$ where $k \le n$
  - STEP 1
    **Divide** the **leading term of the polynomial** $P(x)$ by **the leading term of the divisor** $D(x)$ :
    $$\frac{a_n x^n}{b_b x^k} = q_m x^m$$
  - STEP 2
    **Multiply the divisor** by this term: $D(x) \times q_m x^m$
  - STEP 3
    **Subtract this** from the **original polynomial** $P(x)$ to cancel out the leading term:
    $$R(x) = P(x) - D(x) \times q_m x^m$$
  - Repeat steps 1 – 3 using the new polynomial $R(x)$ in place of $P(x)$ until the subtraction results in an expression for $R(x)$ with degree less than the divisor
    - The quotient $Q(x)$ is the **sum of the terms** you multiplied the divisor by:
      $$Q(x) = q_m x^m + q_{m-1} x^{m-1} + \ldots + q_1 x + q_0$$
    - The remainder $R(x)$ is the polynomial after the final subtraction

### Division by linear functions

- If $P(x)$ has degree $n$ and is divided by a linear function $(ax + b)$ then
  - $\frac{P(x)}{ax + b} = Q(x) + \frac{R}{ax + b}$ where
    - $ax + b$ is the **divisor** (degree 1)
    - $Q(x)$ is the **quotient** (degree $n$ – 1)
    - $R$ is the **remainder** (degree 0)
  - Note that $P(x) = Q(x) \times (ax + b) + R$

Your notes

## Division by quadratic functions

- If $P(x)$ has degree $n$ and is divided by a quadratic function $(ax^2 + bx + c)$ then
  - $\frac{P(x)}{ax^2 + bx + c} = Q(x) + \frac{ex + f}{ax^2 + bx + c}$ where
    - $ax^2 + bx + c$ is the **divisor** (degree 2)
    - $Q(x)$ is the **quotient** (degree $n - 2$)
    - $ex + f$ is the **remainder** (degree less than 2)
  - The remainder will be **linear** (degree 1) if $e \neq 0$, and **constant** (degree 0) if $e = 0$
  - Note that $P(x) = Q(x) \times (ax^2 + bx + c) + ex + f$

## Division by polynomials of degree *k* ≤ *n*

- If $P(x)$ has degree $n$ and is divided by a polynomial $D(x)$ with degree $k \leq n$
  - $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$ where
    - $D(x)$ is the **divisor** (degree $k$)
    - $Q(x)$ is the **quotient** (degree $n - k$)
    - $R(x)$ is the **remainder** (degree less than $k$)
  - Note that $P(x) = Q(x) \times D(x) + R(x)$

## Are there other methods for dividing polynomials?

- **Synthetic division** is a faster and shorter way of setting out a division when dividing by a linear term of the form
  - To divide $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ by $(x - c)$:
    - Set $b_n = a_n$
    - Calculate $b_{n-1} = a_{n-1} + c \times b_n$
    - Continue this iterative process $b_{i-1} = a_{i-1} + c \times a_i$
    - The quotient is $Q(x) = b_n x^{n-1} + b_{n-1} x^{n-2} + \ldots + b_2 x + b_1$ and the remainder is $r = b_0$
- You can also find quotients and remainders by **comparing coefficients**
  - Given a polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$
  - And a divisor $D(x) = d_k x^k + d_{k-1} x^{k-1} + \ldots + d_1 x + d_0$
  - Write $Q(x) = q_{n-k} x^{n-k} + \ldots + q_1 x + q_0$ and $R(x) = r_{k-1} x^{k-1} + \ldots + r_1 x + r_0$
  - Write $P(x) = Q(x)D(x) + R(x)$
    - Expand the right-hand side
    - Equate the coefficients
    - Solve to find the unknowns $q$'s & $r$'s

### Examiner Tip

- In an exam you can use whichever method to divide polynomials - just make sure your method is written clearly so that if you make a mistake you can still get a mark for your method!

Your notes

Your notes

## Worked example

a) Perform the division $\dfrac{x^4 + 11x^2 - 1}{x+3}$. Hence write $x^4 + 11x^2 - 1$ in the form $Q(x) \times (x+3) + R$.

Step 1: what do we multiply $x$ by to get $x^4$?

$$\begin{array}{r|l} & x^3 \\ \hline x+3 & x^4 + 0x^3 + 11x^2 + 0x - 1 \end{array}$$

Note: $0x^3$ and $0x$ are used to keep like terms together.

Step 2: subtract $x^3(x+3) = x^4 + 3x^3$ from $x^4 + 0x^3$

$$\begin{array}{r|l} & x^3 \\ \hline x+3 & x^4 + 0x^3 + 11x^2 + 0x - 1 \\ & -(x^4 + 3x^3) \\ \hline & \quad -3x^3 \end{array}$$

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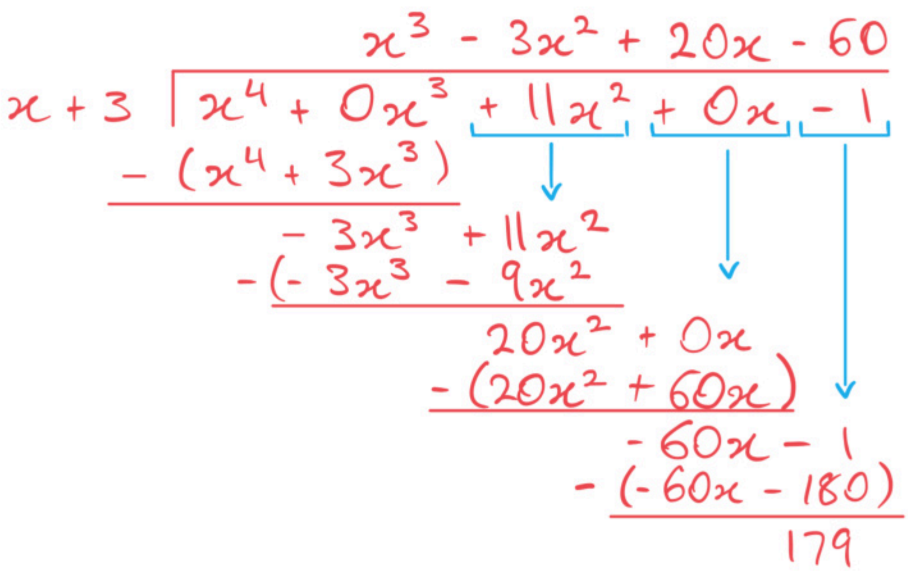
Step 3: bring the $11x^2$ down and return to step 1.



$$x^4 + 11x^2 - 1$$
$$= (x^3 - 3x^2 + 20x - 60)(x + 3) + 179$$

b) Find the quotient and remainder for $\dfrac{x^4 + 4x^3 - x + 1}{x^2 - 2x}$. Hence write $x^4 + 4x^3 - x + 1$ in the form $Q(x) \times (x^2 - 2x) + R(x)$.

Your notes

When dividing by quadratics use the same steps as above.

$$
\begin{array}{r}
x^2 + 6x + 12 \\
x^2 - 2x \,\big)\, x^4 + 4x^3 + 0x^2 - x + 1 \\
-\,(x^4 - 2x^3) \qquad\qquad\qquad \\
6x^3 + 0x^2 \qquad\quad \\
-\,(6x^3 - 12x^2) \qquad\quad \\
12x^2 - x \quad\; \\
-\,(12x^2 - 24x) \quad\; \\
23x + 1
\end{array}
$$

$$x^4 + 4x^3 - x + 1$$
$$= (x^2 + 6x + 12)(x^2 - 2x) + 23x + 1$$

Your notes

## 2.7.3 Polynomial Functions

### Sketching Polynomial Graphs

In exams you'll commonly be asked to sketch the graphs of different polynomial functions with and without the use of your GDC.

#### What's the relationship between a polynomial's degree and its zeros?

- If a **real polynomial** $P(x)$ has **degree $n$**, it will have **$n$ zeros** which can be written in the form $a + bi$, where $a, b \in \mathbb{R}$
  - For example:
    - A quadratic will have 2 zeros
    - A cubic function will have 3 zeros
    - A quartic will have 4 zeros
  - Some of the zeros may be **repeated**
- Every **real polynomial** of **odd degree** has **at least one real zero**

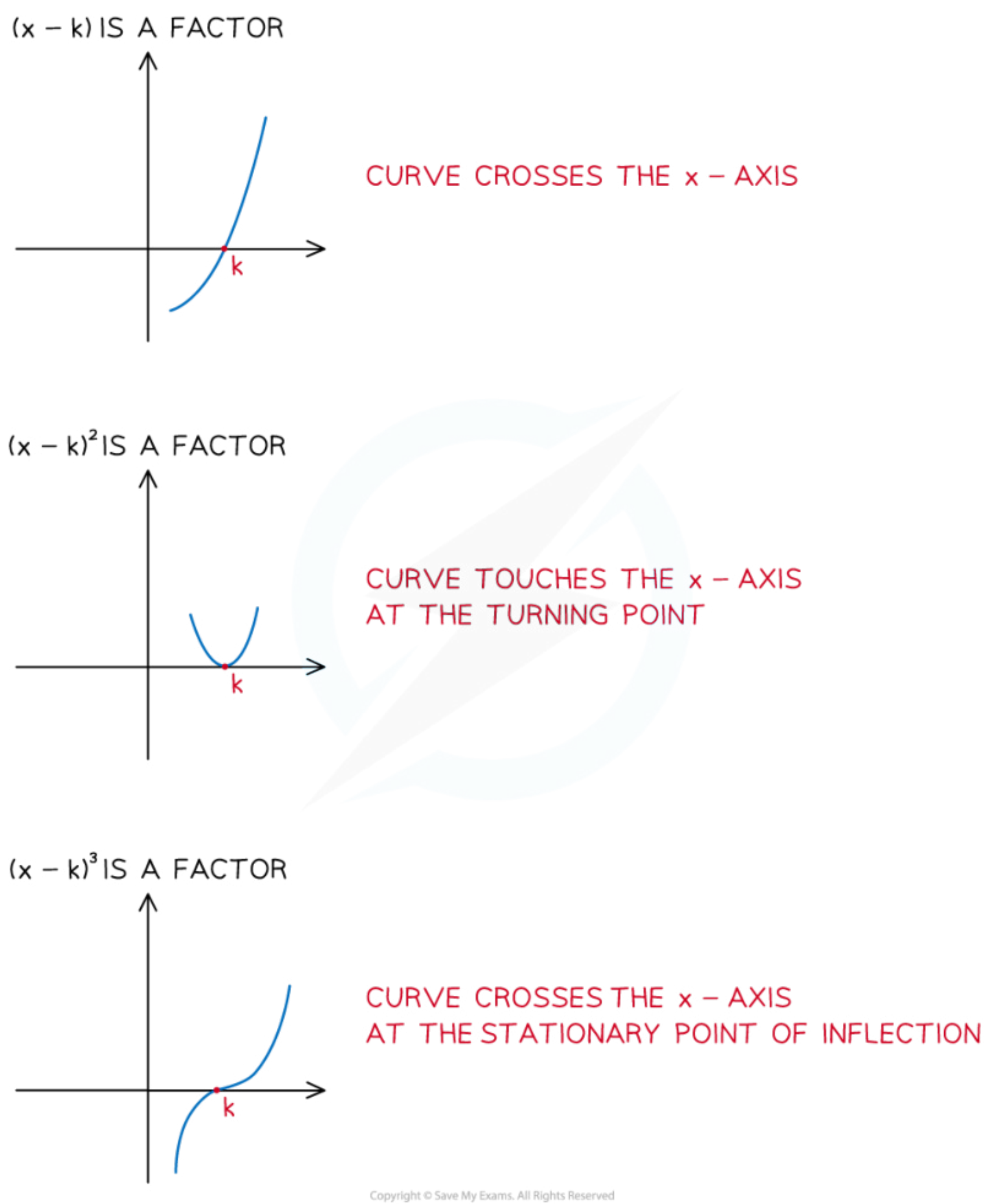
#### How do I sketch the graph of a polynomial function without a GDC?

- Suppose $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ ... + a_1 x + a_0$ is a **real polynomial** with **degree $n$**
- To sketch the graph of a polynomial you need to know three things:
  - The **$y$-intercept**
    - Find this by **substituting $x$ = 0** to get **$y = a_0$**
  - The **roots**
    - You can find these by **factorising** or solving **$y$ = 0**
  - The **shape**
    - This is determined by the **degree** ($n$) and the sign of the **leading coefficient** ($a_n$)

#### How does the multiplicity of a real root affect the graph of the polynomial?

- The **multiplicity** of a root is the number of times it is **repeated** when the polynomial is factorised
  - If $x = k$ is a root with **multiplicity $m$** then $(x - k)^m$ is a **factor** of the polynomial
- The graph either **crosses** the $x$-axis or **touches** the $x$-axis at a **root $x = k$** where $k$ is a real number
  - If $x = k$ has **multiplicity 1** then the graph **crosses** the $x$-axis at $(k, 0)$
  - If $x = k$ has **multiplicity 2** then the graph has a **turning point** at $(k, 0)$ so **touches** the $x$-axis
    - If $x = k$ has **odd multiplicity** $m \geq 3$ then the graph has a **stationary point of inflection** at $(k, 0)$ so **crosses** the $x$-axis
    - If $x = k$ has **even multiplicity** $m \geq 4$ then the graph has a **turning point** at $(k, 0)$ so **touches** the $x$-axis

Your notes



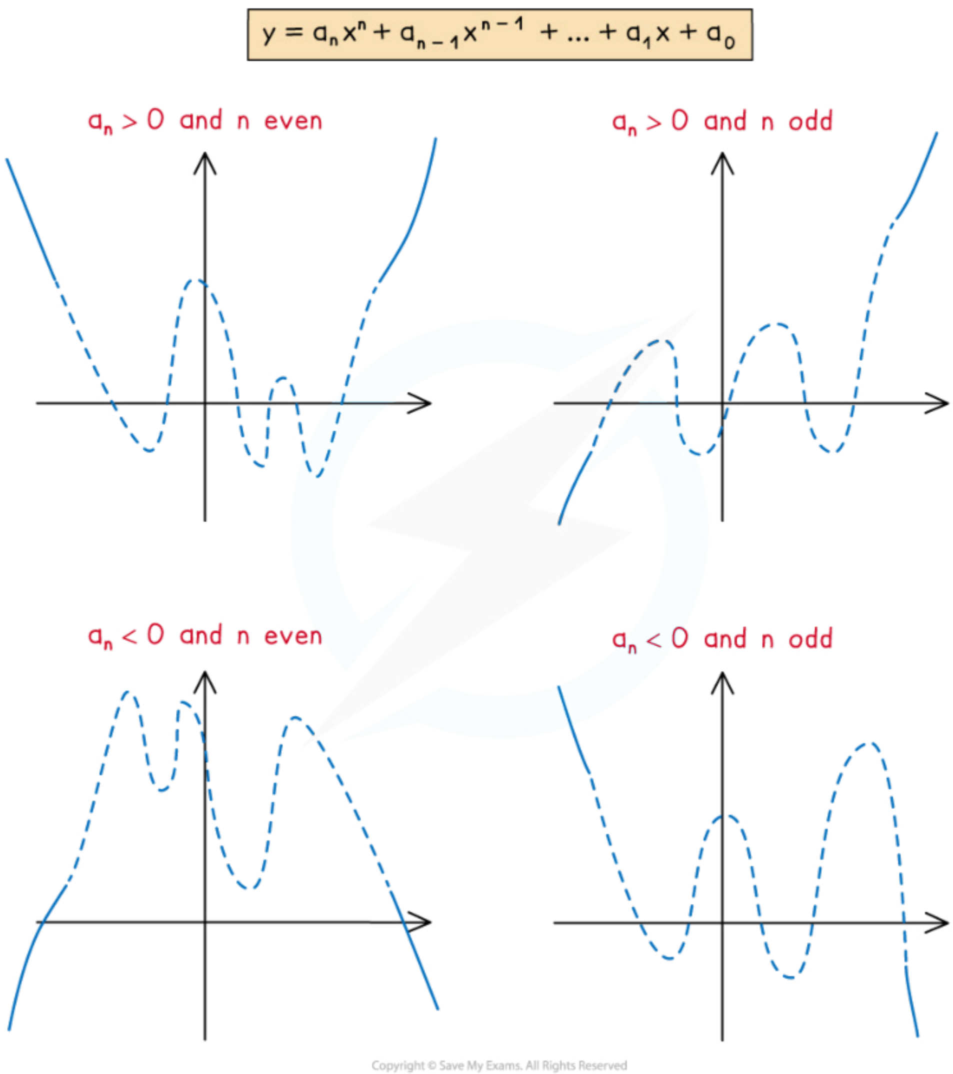

### How do I determine the shape of the graph of the polynomial?

- Consider what happens as ***x* tends to ± ∞**
  - If $a_n$ is **positive** and *n* is **even** then the graph **approaches from the top left** and **tends to the top right**
    - $\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = +\infty$

Your notes

    - If $a_n$ is **negative** and $n$ is **even** then the graph **approaches from the bottom left** and **tends to the bottom right**
        - $\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = +\infty$
    - If $a_n$ is **positive** and $n$ is **odd** then the graph **approaches from the bottom left** and **tends to the top right**
        - $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to +\infty} f(x) = +\infty$
    - If $a_n$ is **negative** and $n$ is **odd** then the graph **approaches from the top left** and **tends to the bottom right**
        - $\lim_{x \to -\infty} f(x) = +\infty$ and $\lim_{x \to +\infty} f(x) = -\infty$
- Once you know the **shape**, the **real roots** and the ***y*-intercept** then you simply connect the points using a **smooth curve**
- There will be **at least one turning point** in-between each pair of roots
    - If the degree is $n$ then there is **at most $n$ – 1 stationary points (**some will be **turning points**)
        - Every real polynomial of **even degree** has **at least one turning point**
        - Every real polynomial of **odd degree bigger than 1** has **at least one point of inflection**
    - If it is a calculator paper then you can use your GDC to find the coordinates of the turning points
    - You won't need to find their location without a GDC unless the question asks you to

Your notes




### Examiner Tip

- If it is a calculator paper then you can use your GDC to find the coordinates of any turning points
- If it is the non-calculator paper then you will not be required to find the turning points when sketching unless specifically asked to

Your notes

## Worked example

a) The function $f$ is defined by $f(x) = (x+1)(2x-1)(x-2)^2$. Sketch the graph of $y = f(x)$.

Find the y-intercept

$x = 0$ : $y = (1)(-1)(-2)^2 = -4$

Find the roots and determine if graphs crosses or touches the x-axis

$(x+1)(2x-1)(x-2)^2$

$(-1, 0)$ $(\frac{1}{2}, 0)$ $(2, 0)$

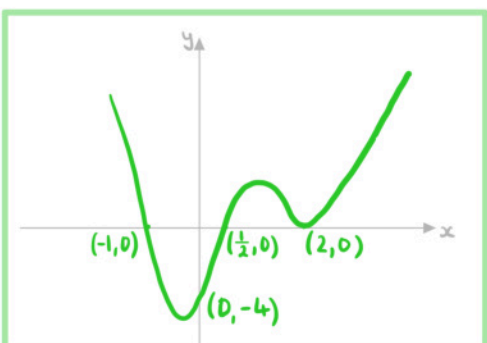
cross cross touch

Determine the shape by looking at the leading term

Leading term is $(x)(2x)(x)^2 = 2x^4$

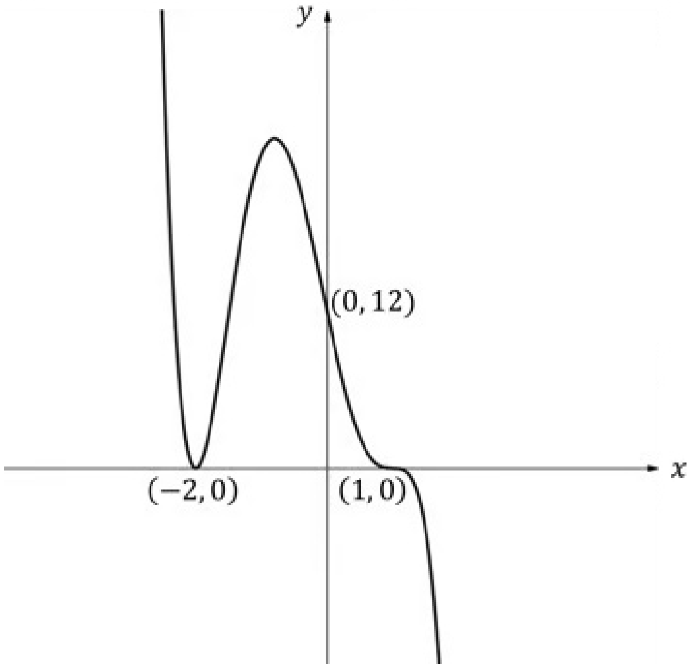
As $x \to -\infty$ $y \to +\infty$

As $x \to +\infty$ $y \to +\infty$




b) The graph below shows a polynomial function. Find a possible equation of the polynomial.

Your notes




Touches at (-2,0) $(x+2)^2$ is a factor

Point of inflection at (1,0) $(x-1)^3$ is a factor

Write in the form of : $y = a(x+2)^2(x-1)^3$

Use the y-intercept to find a

$12 = a(2)^2(-1)^3 \Rightarrow -4a = 12 \quad \therefore a = -3$

$$y = -3(x+2)^2(x-1)^3$$

Your notes

## Solving Polynomial Equations

### What is "The Fundamental Theorem of Algebra"?

- Every **real polynomial** with degree *n* can be factorised into ***n* complex linear factors**
  - Some of which may be **repeated**
  - This means the polynomial will have *n* zeros (some may be repeats)
- Every **real polynomial** can be expressed as a product of **real linear factors** and **real irreducible quadratic factors**
  - An irreducible quadratic is where it **does not have real roots**
    - The **discriminant** will be negative: $b^2 - 4ac < 0$
- If $a + bi$ ($b \neq 0$) is a **zero** of a **real polynomial** then its **complex conjugate** $a - bi$ is also a **zero**
- Every **real polynomial** of **odd degree** will have **at least one real zero**

### How do I solve polynomial equations?

- Suppose you have an equation $P(x) = 0$ where $P(x)$ is a **real polynomial of degree *n***
  - $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ \ldots + a_1 x + a_0$
- You may be given one zero or you might have to find a zero $x = k$ by substituting values into $P(x)$ until it equals 0
- If you know a **root** then you know a **factor**
  - If you know $x = k$ **is a root** then $(x - k)$ **is a factor**
  - If you know $x = a + bi$ **is a root** then you know a **quadratic factor** $(x - (a + bi))(x - (a - bi))$
    - Which can be written as $((x - a) - bi)((x - a) + bi)$ and **expanded quickly using difference of two squares**
- You can then **divide** $P(x)$ by this factor to get **another factor**
  - For example: dividing a cubic by a linear factor will give you a quadratic factor
- You then may be able to factorise this new factor

### Examiner Tip

- If a polynomial has three or less terms check whether a substitution can turn it into a quadratic
  - For example: $x^6 + 3x^3 + 2$ can be written as $(x^3)^2 + 3(x^3) + 2$

Your notes

## Worked example

Given that $x = \frac{1}{2}$ is a zero of the polynomial defined by $f(x) = 2x^3 - 3x^2 + 5x - 2$, find all three zeros of $f$.

$x = \frac{1}{2}$ is a root $\therefore$ $(2x - 1)$ is a factor

Find the quadratic factor $(2x^3 - 3x^2 + 5x - 2) = (2x - 1)(ax^2 + bx + c)$

Compare coefficients : $2x^3 = 2ax^3 \quad \therefore a = 1$

$-2 = -c \quad \therefore c = 2$

$5x = 2cx - bx \Rightarrow 5 = 4 - b \quad \therefore b = -1$

Solve the quadratic : $x^2 - x + 2 = 0$

Formula booklet

| Solutions of a quadratic equation | $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
|---|---|

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-7}}{2} = \frac{1}{2} \pm \frac{\sqrt{7}}{2}i$$

**Roots : $\frac{1}{2}$, $\frac{1}{2} + \frac{\sqrt{7}}{2}i$, $\frac{1}{2} - \frac{\sqrt{7}}{2}i$**

# 2.7.4 Roots of Polynomials

Your notes

## Sum & Product of Roots

### How do I find the sum & product of roots of polynomials?

- Suppose $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ \dots + a_1 x + a_0$ is a **polynomial** of **degree** *n* with *n* roots $\alpha_1, \alpha_2, \dots, \alpha_n$
  - The polynomial is written as $\sum_{r=0}^{n} a_r x^r = 0,\ a_n \neq 0$ in the **formula booklet**
  - $a_n$ is the coefficient of the **leading term**
  - $a_{n-1}$ is the coefficient of the $\mathbf{x^{n-1}}$ **term**
    - Be careful: this could be equal to zero
  - $a_0$ is the **constant term**
    - Be careful: this could be equal to zero
- In factorised form: $P(x) = a_n(x - \alpha_1)(x - \alpha_2)\dots(x - \alpha_n)$
  - Comparing coefficients of the $\mathbf{x^{n-1}}$ **term** and the **constant term** gives
    - $a_{n-1} = a_n(-\alpha_1 - \alpha_2 - \dots - \alpha_n)$
    - $a_0 = a_n(-\alpha_1) \times (-\alpha_2) \times \dots \times (-\alpha_n)$
- The **sum** of the roots is given by:
  - $\alpha_1 + \alpha_2 + \ \dots + \alpha_n = -\dfrac{a_{n-1}}{a_n}$
- The **product** of the roots is given by:
  - $\alpha_1 \times \alpha_2 \times \ \dots \times \alpha_n = \dfrac{(-1)^n a_0}{a_n}$
    - both of these formulae are in your **formula booklet**

### How can I find unknowns if I am given the sum and/or product of the roots of a polynomial?

- If you know a complex root of a real polynomial then its **complex conjugate** is **another root**
- Form **two equations** using the roots
  - One using the **sum of the roots formula**
  - One using the **product of the roots formula**
- **Solve** for any unknowns

## Examiner Tip

- Examiners might trick you by not having an $x^{n-1}$ term or a constant term
- To make sure you do not get tricked you can write out the full polynomial using 0 as a coefficient where needed
  - For example: Write $x^4 + 2x^2 - 5x$ as $x^4 + 0x^3 + 2x^2 - 5x + 0$

Your notes

Your notes

## Worked example

$2-3\mathrm{i}$, $\frac{5}{3}\mathrm{i}$ and $\alpha$ are three roots of the equation $18x^5 - 9x^4 + 32x^3 + 794x^2 - 50x + k = 0$

.

a) Use the sum of all the roots to find the value of $\alpha$.

It is a real polynomial so if $a+bi$ is a root then $a-bi$ is also a root

Roots: $2-3i$, $2+3i$, $\frac{5}{3}i$, $-\frac{5}{3}i$, $\alpha$

Formula booklet

| | |
|---|---|
| Sum & product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$ | Sum is $\frac{-a_{n-1}}{a_n}$ |

$18x^5 - 9x^4 + 32x^3 + 794x^2 - 50x + k$

$a_n = 18 \quad a_{n-1} = -9$

$(2-3i)+(2+3i)+\left(\frac{5}{3}i\right)+\left(-\frac{5}{3}i\right)+\alpha = \frac{-(-9)}{18}$

$4 + \alpha = \frac{1}{2}$

$\alpha = -\frac{7}{2}$

b) Use the product of all the roots to find the value of $k$.

Formula booklet

| | |
|---|---|
| Sum & product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$ | product is $\frac{(-1)^n a_0}{a_n}$ |

$18x^5 - 9x^4 + 32x^3 + 794x^2 - 50x + k$

$a_n = 18 \quad n = 5 \quad a_0 = k$

$(2-3i)(2+3i)\left(\frac{5}{3}i\right)\left(-\frac{5}{3}i\right)\left(-\frac{7}{2}\right) = \frac{(-1)^5 k}{18}$

$(13)\left(\frac{25}{9}\right)\left(-\frac{7}{2}\right) = \frac{-k}{18}$

$-\frac{2275}{18} = -\frac{k}{18}$

$k = 2275$